AP Calculus BC Course Description

The AP Calculus BC course is a one semester class meeting for 90 days, 90 minutes per day. Students who enroll in BC in the spring have taken and passed AP Calculus AB during the fall semester, and generally go on to take the AP Calculus BC Exam. Conceptual understanding is emphasized, and there is an essential balance in the course between technology and pencil and paper calculations. Each student has his or her own TI-84 plus calculator issued by the school to use for the duration of the course.

Through lecture, independent exploration, practice and collaboration, the students gain an in-depth understanding of the material and make valuable connections between and among topics. At the beginning of each class, the students work together on an introductory problem or activity, discussing ideas and approaches to reach a solution. Students are encouraged to ask questions and engage in class discussions. Homework is collected weekly and graded for completeness and correctness. Students are expected complete out of class assignments and may collaborate on them at the beginning of the next class, allowing students the opportunity to communicate mathematics verbally on a daily basis.

Note: The following outline contains both semesters of AP Calculus. Calculus BC topics that are covered in the first semester are denoted by an asterisk (*). Calculus BC topics that are covered in the second semester are denoted by a double asterisk (**).

Course Outline

I. Unit 1: Functions and Graphs (Precalculus Review) 6 Days
   a. Linear Functions
   b. Exponential Functions
   c. Logarithmic Functions
   d. Inverse Functions
   e. Parametric Curves
   f. Trigonometric Functions

Teaching Notes:

The focus is on the analysis of graphs and characteristics of functions. Local and global behavior of functions is considered numerically, analytically, and by utilizing the support of graphing technology. Students are expected to make connections between the various representations. Also, the importance of considering all of the representations is emphasized. For example, students are expected to realize that graphical representation is not always sufficient to describe characteristics of functions (e.g. removable discontinuity). From the very beginning of the course, students are encouraged to formulate mathematically sound arguments and reasonable solutions in both verbal and written form.

II. Unit 2: Limits and Continuity (10 Days)
   a. Limits
      i. Informal definition of Limits and Notation
      ii. Properties of Limits
      iii. Formal Definition of Limit
      iv. Limits Involving Infinity
      v. Asymptotic Behavior
   b. Definition of Continuity
      i. Properties of continuity
      ii. Continuous extension
      iii. Intermediate Value Theorem
   c. Tangent Lines
      i. Average Rate of Change
      ii. Instantaneous Rate of Change
      iii. Tangents and Normal lines

Teaching Notes:

Limits are introduced using a graphical, numerical and analytical approach. One-sided limits are also evaluated and explored. Early on, the concept of infinity is incorporated through end behavior and asymptotic behavior of graphs. A strong emphasis is placed on reasons why a limit fails to exist. Graphical exploration is followed by analytical interpretation. Students use the Squeeze (Sandwich) Theorem to solidify their understanding of limits and begin their work with proofs. Verbal or written explanation is required along with their proof.

Continuity is initially approached from a graphical perspective. The idea of proof is revisited with the Intermediate Value Theorem. Through the understanding of local linearity and limits, students construct the concept of the tangent line.

III. Derivatives (14 Days)
   a. Definition of Derivative
   b. Differentiability
   c. Introduction to the relationship between the graphs of f and f'
   d. Differentiation Rules
   e. Rates of Change
   f. Derivatives of Trigonometric Functions
   g. Chain Rule
   h. Parametric Differentiation
   i. Implicit Differentiation

j. Derivatives of Exponential and Logarithmic Functions
k. Derivatives of Inverse Trigonometric Functions

Teaching Notes:

In the beginning of the unit, major emphasis is placed on the relationship between differentiability and continuity. Connections are made between the derivative, tangent line, slope of curve, average rate of change, and instantaneous rate of change. A major emphasis is placed on local linearity, requiring students to discuss the concept in their own words. Graphical exploration is utilized to "zoom-in" on various graphs to determine differentiability. Once conceptual understanding has been accomplished, the focus is shifted to computing derivatives and rules and techniques for differentiation. The differentiation rules are initially explored through the use of graphing technology. Students make conjectures about derivatives of polynomial, trigonometric and exponential functions. After exploration, many of the derivatives of trigonometric, exponential, logarithmic and inverse trigonometric functions are derived by the students using proofs. In particular, proofs of inverse trigonometric functions are written and verbally presented to the class by student teams.

IV. Applications of the Derivative (15 days)
   a. Mean and Extreme Value Theorem
   b. Increasing and Decreasing intervals
   c. Concavity
   d. Connecting f with f' and f''
   e. Modeling and Optimization
   f. Linearization
   g. Differentials
   h. Related Rates

Teaching Notes:

This unit is built on two major concepts. In the beginning of the unit, the emphasis is for students to make connections between a function and its first and second derivatives. This is done graphically, algebraically, and numerically. Also, the concept of the derivative as a rate of change is reinforced with reference to position, velocity and acceleration.

The second part of the unit focuses on the application of the derivative to solving problems, such as optimization and related rates problems. Students solve problems analytically, supporting their conclusions with the use of graphing technology. Students gain an appreciation for the usefulness of calculus through real-world scenarios. Also, the concept of approximating functions using tangent lines and differentials is explored graphically, numerically and analytically.

V. Integrals (14 Days)
   a. Riemann Sums
      i. Right

        ii. Left
        iii. Midpoint
   b. Definite Integrals
   c. Fundamental Theorem of Calculus
   d. Trapezoidal Approximation of Definite Integrals
   e. Indefinite Integrals
   f. Integration Techniques
        i. Integration by Substitution*
        ii. Integration by Parts and Tabular Integration*

Teaching Notes:

During this unit, there is a strong emphasis of the concept of a definite integral as a numerical value. Approximations involving Riemann Sums and Trapezoids are explored and compared. Right, Left, and Midpoint Riemann Sums and Trapezoidal approximations are done by hand and using programs for the graphing calculator. Students look at indefinite integrals as mathematical objects, mainly general antiderivatives. Integration properties, formulas and techniques are introduced and practiced. In particular, integration by substitution is carried out both by changing the limits of integration in the substitiution and also returning to the original expression and using the given limits of integration. Relationships between functions and their antiderivatives are explored using graphing technology, particularly the relationship between position, velocity, and acceleration. One of the major concepts of this unit is the Fundamental Theorem of Calculus, Parts 1 and 2. Students are exposed to a variety of problems involving the FTC and its applications. Students are given excerpts from the College Board's AP Calculus 2005-2006 Professional Development Workshop Materials <u>Special Focus: The Fundamental Theorem of Calculus</u>. Students work collaboratively to reach a deep level of understanding of the FTC, teaching students how to communicate mathematics verbally. Released AB Free Response questions are used to facilitate proper written explanation of mathematical concepts.

VI. Applications of Definite and Indefinite Integrals (22 days)
   a. Differential Equations
   b. Slope Fields
   c. Separable Differential Equations
   d. Law of Exponential Change
   e. Euler's Method*
   f. Partial Fraction Decomposition*
   g. Integration by Partial Fractions*
   h. Logistic Differential equations*
   i. Integrals as Net Change
   j. Areas in the Plane
        i. Area under curve

ii. Area between curves

k. Lengths of Curves (including parametric)*

l. Volumes

i. Disc method

ii. Washer method

iii. Shell method

iv. Cross-sectional area

Teaching Notes:

In the first part of this unit, the focus is on differential equations, using slope fields as a method to find solution curves and Euler's method as a numerical approximation of the function's value. Students draw slope fields by hand and use a slope field program for the graphing calculators. Discussion is facilitated by use of the overhead projection capabilities of the graphing calculator. Calculations for Euler's method are done by hand using step-by-step iteration and using a calculator program. Students develop written and verbal arguments for whether the solution produced by Euler's method is an over- or under-estimate of the actual solution based on both analytical and graphical analysis. After students have obtained a solid understanding of the concept of a differential equation, separable differential equations are introduced and exponential growth and decay are emphasized.

The second part of the unit focuses on applications of integrals as an accumulation of a rate of change, distance, area and volume. Formulas for lengths of curves are introduced. Students make geometric connections between curve lengths of circles in function and parametric form using circumference. As an introduction to volume, computer-based solids of revolutions are explored at http://college.hmco.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html. Volume is calculated using the disc, washer and shell methods, and using a variety of known cross-sectional areas.

VII. Improper Integrals** (10 days)

a. L'Hopital's Rule and Indeterminate Form

b. Relative Growth Rates

c. Improper Integrals

i. Infinite limits of integration

ii. Discontinuities

iii. Applications

iv. Comparison Test

Teaching Notes:

In the beginning of the unit, students explore limits of functions having indeterminate forms graphically and numerically. The concept of indeterminate form is discussed. Limits are calculated analytically using L'Hopital's Rule. Improper integrals are explored as limits of definite integrals. Several functions with horizontal asymptotes or infinite discontinuities are integrated analytically and students are encouraged to discuss the concept of integrating to infinity. The method of partial fractions and L'Hopital's Rule is used in the evaluation of improper integrals, when necessary. The Comparison test for convergence or divergence of improper integrals is used, with support from graphing technology. Solids with infinite surface area and finite volume (such as 'Gabriel's Horn') are used to enhance the application of improper integrals.

VIII. Polynomial Approximations and Series** (25 days)
- a. Sequences
  - i. Explicitly defined
  - ii. Recursively defined
  - iii. Arithmetic
  - iv. Geometric
  - v. Graphing
- b. Series
- c. Power Series
- d. Taylor and Maclaurin Series
  - i. Manipulating Taylor and Maclaurin series as algebraic objects
  - ii. Approximating polynomials using series
  - iii. Error (including Lagrange Error Bound)
- e. Convergence Tests
  - i. Radius and Interval of Convergence
  - ii. n^{th} term test
  - iii. comparison test
  - iv. integral test
  - v. ratio test
  - vi. alternating series test
  - vii. p-series

Teaching Notes:

This unit starts out by taking a look at sequences. Students use formulas to find terms and write explicit and recursive formulas from a list of terms. In particular, arithmetic and geometric sequences are studied. Graphing calculators are used to graph sequences in parametric mode (for explicit formulas) and in sequence mode (for recursive formulas).

After an understanding of sequences is developed, the concept of a series is introduced. Students construct a sequence of partial sums to determine convergence or divergence of the series. Convergence is determined numerically, graphically or analytically using a formula for the n^{th} partial sum. Students express repeating decimals as an infinite series. The study of series continues with power series. In particular, geometric power series and their convergence or divergence is explored. Graphing calculators support the idea of representing a function with a power series and the interval of convergence. Power series are manipulated as algebraic objects, using term-by-term differentiation and integration. Again, conclusions are supported graphically.

In the next part of the unit, polynomial approximation using Taylor and Maclaurin series is emphasized. Students use the general formula for writing a Taylor series and explore the approximation numerically and graphically. Error is calculated analytically and graphically, and by using the Lagrange error bound. Students become very familiar with the Taylor series for $\sin x$, $\cos x$, and e^x. They perform manipulations on Taylor series, including differentiation, integration, substitution, addition, and subtraction.

The last part of the unit emphasizes convergence of series. The radius and interval of convergence are determined analytically and supported graphically. The discussion begins with the n^{th} term test and geometric, harmonic, alternating, and p-series. The ratio test is used to expand the idea to other power series. Convergence is also determined using integral and comparison tests. Throughout this part of the unit, students are encouraged to work collaboratively to construct their arguments for convergence or divergence. They express arguments verbally and in writing.

IX. Parametric, Polar and Vector Functions** (12 days)
   a. Parametric Equations
   b. Polar Coordinates
   c. Slope of Polar Curves
   d. Area of Polar Region
   e. Vectors in the Plane
   f. Calculus of Vectors
      i. Derivatives of Vector Functions
      ii. Position, Velocity, Acceleration
      iii. Speed
      iv. Displacement
      v. Distance Traveled

Teaching Notes:

Characteristics of Parametric, Polar and Vector functions are studied and analyzed. In particular, students explore the different representations of circles and lines in all forms to make comparisons. Graphing technology is used to enhance the study of parametric curves, modeling direction of motion and paths of particles. Students explore particle motion using simultaneous graphing capabilities and analytic analysis.

Polar coordinates are introduced for basic understanding and some graphing is done by hand. Students explore different types of polar curves through a graphing calculator activity involving cardiods, limacons, rose curves, etc... They work collaboratively to make connections between the equation and the type of graph that is produced, communicating mathematics verbally and in writing. After a basic understanding of polar functions is established, derivatives and integrals of polar functions are studied. Slope of a polar curve and area of a polar region is considered. Emphasis is placed on limits of integration.

Vector functions are introduced with an emphasis on application and connections to parametric functions. Navigation, position, velocity, acceleration, and speed problems are explored. Derivatives and integrals of vector functions are considered using application problems to motivate the teaching. Students make connections between distance traveled by a particle in vector form and the formula for curve length of a parametric curve.

X. AP Exam Review (25 days)
   a. Workbooks
   b. Free Response
   c. Released Exams
   d. Practice Exams
   e. Tips and Strategies

Teaching Notes:

Following the completion of the curriculum, students are involved in an extensive review for the AP Calculus BC exam. Throughout the review, students work collaboratively in order to strengthen their verbal and written mathematical communication and construction of arguments. They also work independently in order to evaluate their personal strengths and weaknesses. Graphing calculators are used to support conclusions and solve problems. Test taking tips and strategies are shared, discussed and practiced. Published practice books and released College Board materials are utilized during the review. Formulas and theorems are reviewed and committed to memory.

Assessments

Semester One (AB):

Exam: Chapter 1 Quiz: Definite Integrals

Quiz: Section 2.1, 2.2	Exam: Chapter 5
Exam: Chapter 2	Quiz: Section 6.2, 6.3
Quiz: Sections 3.5 – 3.9	Quiz: Section 6.1, 6.4
Exam: Chapter 3	Exam: Chapter 6
Quiz: Section 4.1 – 4.4	Quiz: Section 7.1, 7.2, 7.4
Quiz: Section 4.5, 4.6	Exam: Chapter 7
Exam: Chapter 4	Final Exam

Semester Two (BC):

Exam: Chapter 8	AP Free Response Questions
Exam: Chapter 9	Released Exams
Exam: Chapter 10	Formula and Theorem Quizzes (3)

In addition to teacher-generated quizzes and exams, students are also exposed to released Free-response questions from past AB exams and multiple choice practice problems written for AP Exam preparation. These problems are chosen and/or written to facilitate written student explanation of concepts.

Primary textbook

Finney, Demana, Waits, Kennedy
Calculus: Graphical, Numerical, Algebraic (Third Edition) AP Edition
Pearson Prentice Hall. Boston, Massachusetts, 2007.

Supplemental Resources

Textbooks and Workbooks:

1. Barton, Brunsting, Diehl, Hill, Tyler, Wilson
Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text)
Pearson Education. Boston, Massachusetts, 2007.

2. Larson, Hostetler, Edwards
Calculus (Sixth edition)
Houghton Mifflin Company. Boston, Massachusetts, 1998.

3. Lederman, David
Multiple Choice & Free Response Questions in Preparation for the AP Calculus BC Examination (Sixth Edition)
D&S Marketing Systems, Inc. Brooklyn, New York, 1998.

4. Arterburn, Hubbard, Perl
The Best Test Preparation for the Advanced Placement Examination:
Mathematics Calculus BC
Research and Education Association. Piscataway, NJ, 1997.

Websites:

1. AP Central

http://apcentral.collegeboard.com/apc/Controller.jpf

2. Houghton Mifflin: Calculus with Analytic Geometry

http://college.hmco.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html.